

Contents

Preface	<i>xi</i>
Symbols and Abbreviations	<i>xv</i>
1 Introduction	<i>1</i>
2 Mathematical Foundations	<i>13</i>
2.1 Matrix Algebra	<i>13</i>
2.2 Vector Algebra	<i>20</i>
2.3 Simultaneous Linear Equation Systems	<i>22</i>
2.4 Linear Dependence	<i>26</i>
2.5 Convex Sets and n -Dimensional Geometry	<i>29</i>
3 Introduction to Linear Programming	<i>35</i>
3.1 Canonical and Standard Forms	<i>35</i>
3.2 A Graphical Solution to the Linear Programming Problem	<i>37</i>
3.3 Properties of the Feasible Region	<i>38</i>
3.4 Existence and Location of Optimal Solutions	<i>38</i>
3.5 Basic Feasible and Extreme Point Solutions	<i>39</i>
3.6 Solutions and Requirement Spaces	<i>41</i>
4 Computational Aspects of Linear Programming	<i>43</i>
4.1 The Simplex Method	<i>43</i>
4.2 Improving a Basic Feasible Solution	<i>48</i>
4.3 Degenerate Basic Feasible Solutions	<i>66</i>
4.4 Summary of the Simplex Method	<i>69</i>
5 Variations of the Standard Simplex Routine	<i>71</i>
5.1 The M -Penalty Method	<i>71</i>
5.2 Inconsistency and Redundancy	<i>78</i>
5.3 Minimization of the Objective Function	<i>85</i>

5.4	Unrestricted Variables	86
5.5	The Two-Phase Method	87
6	Duality Theory	95
6.1	The Symmetric Dual	95
6.2	Unsymmetric Duals	97
6.3	Duality Theorems	100
6.4	Constructing the Dual Solution	106
6.5	Dual Simplex Method	113
6.6	Computational Aspects of the Dual Simplex Method	114
6.7	Summary of the Dual Simplex Method	121
7	Linear Programming and the Theory of the Firm	123
7.1	The Technology of the Firm	123
7.2	The Single-Process Production Function	125
7.3	The Multiactivity Production Function	129
7.4	The Single-Activity Profit Maximization Model	139
7.5	The Multiactivity Profit Maximization Model	143
7.6	Profit Indifference Curves	146
7.7	Activity Levels Interpreted as Individual Product Levels	148
7.8	The Simplex Method as an Internal Resource Allocation Process	155
7.9	The Dual Simplex Method as an Internalized Resource Allocation Process	157
7.10	A Generalized Multiactivity Profit-Maximization Model	157
7.11	Factor Learning and the Optimum Product-Mix Model	161
7.12	Joint Production Processes	165
7.13	The Single-Process Product Transformation Function	167
7.14	The Multiactivity Joint-Production Model	171
7.15	Joint Production and Cost Minimization	180
7.16	Cost Indifference Curves	184
7.17	Activity Levels Interpreted as Individual Resource Levels	186
8	Sensitivity Analysis	195
8.1	Introduction	195
8.2	Sensitivity Analysis	195
8.2.1	Changing an Objective Function Coefficient	196
8.2.2	Changing a Component of the Requirements Vector	200
8.2.3	Changing a Component of the Coefficient Matrix	202
8.3	Summary of Sensitivity Effects	209
9	Analyzing Structural Changes	217
9.1	Introduction	217
9.2	Addition of a New Variable	217

9.3	Addition of a New Structural Constraint	219
9.4	Deletion of a Variable	223
9.5	Deletion of a Structural Constraint	223
10	Parametric Programming	227
10.1	Introduction	227
10.2	Parametric Analysis	227
10.2.1	Parametrizing the Objective Function	228
10.2.2	Parametrizing the Requirements Vector	236
10.2.3	Parametrizing an Activity Vector	245
10.A	Updating the Basis Inverse	256
11	Parametric Programming and the Theory of the Firm	257
11.1	The Supply Function for the Output of an Activity (or for an Individual Product)	257
11.2	The Demand Function for a Variable Input	262
11.3	The Marginal (Net) Revenue Productivity Function for an Input	269
11.4	The Marginal Cost Function for an Activity (or Individual Product)	276
11.5	Minimizing the Cost of Producing a Given Output	284
11.6	Determination of Marginal Productivity, Average Productivity, Marginal Cost, and Average Cost Functions	286
12	Duality Revisited	297
12.1	Introduction	297
12.2	A Reformulation of the Primal and Dual Problems	297
12.3	Lagrangian Saddle Points	311
12.4	Duality and Complementary Slackness Theorems	315
13	Simplex-Based Methods of Optimization	321
13.1	Introduction	321
13.2	Quadratic Programming	321
13.3	Dual Quadratic Programs	325
13.4	Complementary Pivot Method	329
13.5	Quadratic Programming and Activity Analysis	335
13.6	Linear Fractional Functional Programming	338
13.7	Duality in Linear Fractional Functional Programming	347
13.8	Resource Allocation with a Fractional Objective	353
13.9	Game Theory and Linear Programming	356
13.9.1	Introduction	356
13.9.2	Matrix Games	357
13.9.3	Transformation of a Matrix Game to a Linear Program	361
13.A	Quadratic Forms	363

- 13.A.1 General Structure 363
- 13.A.2 Symmetric Quadratic Forms 366
- 13.A.3 Classification of Quadratic Forms 367
- 13.A.4 Necessary Conditions for the Definiteness and Semi-Definiteness of Quadratic Forms 368
- 13.A.5 Necessary and Sufficient Conditions for the Definiteness and Semi-Definiteness of Quadratic Forms 369

14 Data Envelopment Analysis (DEA) 373

- 14.1 Introduction 373
- 14.2 Set Theoretic Representation of a Production Technology 374
- 14.3 Output and Input Distance Functions 377
- 14.4 Technical and Allocative Efficiency 379
 - 14.4.1 Measuring Technical Efficiency 379
 - 14.4.2 Allocative, Cost, and Revenue Efficiency 382
- 14.5 Data Envelopment Analysis (DEA) Modeling 385
- 14.6 The Production Correspondence 386
- 14.7 Input-Oriented DEA Model under CRS 387
- 14.8 Input and Output Slack Variables 390
- 14.9 Modeling VRS 398
 - 14.9.1 The Basic BCC (1984) DEA Model 398
 - 14.9.2 Solving the BCC (1984) Model 400
 - 14.9.3 BCC (1984) Returns to Scale 401
- 14.10 Output-Oriented DEA Models 402

References and Suggested Reading 405

Index 411