

Index

Page references to Figures are followed by the letter ‘f’, references to Tables by the letter ‘t’, while references to Footnotes are followed by the letter ‘n’

a

- accounting loss 141, 183
 - activity levels 150, 189
 - optimal accounting loss
 - figures 5, 10
- activity levels 8, 124, 284
 - interpreted as individual product levels 148–155
 - interpreted as individual resource levels 186–193
 - optimal/optimal output 4, 9
- activity vector
 - changing component of 210–215
 - multiactivity production function 129
 - parameterizing 245–256
- activity/activities
 - see also* activity levels;
 - activity vector
 - additive 129
 - composite 129, 132–133, 158
 - independent 130
 - input 1–2
 - marginal cost function
 - for 276–284, 282t
 - output 6, 7, 183–185, 187, 190, 191
 - composite output activity 171
 - multiactivity joint-production model 171–174
 - output activity mix 189
 - output activity vector 165
 - production 1, 4, 6, 123
 - quadratic programming and
 - activity analysis 335–338
 - shadow 181
 - simple 123, 130
 - supply function for the output
 - of 257–262
- additive activity 129
- additive identity 20
- additive inverse 20
- admissible solutions 298
- Alder, G. 162
- algebra
 - matrix 13–20
 - vector 20–22
- Allen, R.G.D. 123
- allocative efficiency (AE) 380, 383–384
- almost complementary basic solution 330–332

- artificial augmented structural constraint system 72
 - artificial linear programming problem
 - inconsistency and redundancy 78, 81, 83
 - M*-Penalty method 73, 75–77
 - artificial objective function 74
 - artificial variables 73, 93, 111
 - complementary pivot method 330, 331
 - inconsistency and redundancy 79, 84
 - linear fractional functional programming 352
 - two-phase method 88
 - artificial vectors
 - inconsistency and redundancy 78, 79, 81, 82
 - M*-Penalty method 73, 75
 - two-phase method 91, 93
 - associative law
 - matrix algebra 14, 15
 - vector algebra 20
 - augmented linear programming problem 36
 - augmented matrix 23, 24
 - augmented structural constraint system 36, 39, 53
 - artificial 72
 - primal 155–156
 - average cost function
 - average (real) resource cost function 168, 169f, 179f
 - determining 286–295
 - average product 136f, 290, 291–292
 - average product function 139f
 - average productivity
 - average productivity function 127, 138
 - determining 286–295
 - average profit 2, 354
 - average variable cost 142, 292
- b**
- back-substitution 25
 - Banker, R.D. 373–374, 385–386, 398–402
 - basic feasible solutions *see* feasible solutions
 - basic solution 28
 - basic variables 39, 156
 - see also* nonbasic variables
 - complementary pivot method 333
 - computational aspects 43–48, 68, 69
 - improving basic feasible solutions 50, 51, 53–55, 58–60, 62–66
 - duality theory 106, 121
 - dual simplex method 113, 115, 117, 118, 120
 - inconsistency and redundancy 80, 81, 83, 84
 - M*-Penalty method 73, 76, 77
 - parametric programming 229, 237, 239, 255, 272, 274, 278, 281, 287
 - sensitivity analysis 200, 202, 203, 205, 213
 - simplex-based optimization methods 331–334, 340–344, 355
 - structural changes 223, 226
 - two-phase method 87, 90
 - basis for \mathcal{E}^m 27
 - basis inverse, updating 256
 - basis matrix *B* 39, 49, 201, 203
 - Baumol, W.J. 123, 146–148, 311–315, 338
 - BCC (Banker, Charnes and Cooper) model
 - basic 398–399

- input-oriented 399
 - projection 400, 402
 - returns to scale 401–402
 - solving 400
 - Belinski, M. 311–315, 338
 - best-practice extremal frontier 373
 - binding constraints 37
 - Bitran, C. 345, 346, 355
 - boundary point 29
 - Bram, J. 326
- C**
- canonical forms 35–36
 - primal problem 95, 97
 - capacity, excess 4
 - Cauchy-Schwarz inequality 21
 - certainty 1, 6
 - Chadha, S. 347–353
 - Charnes, A. 338, 373–374, 385–390
 - closed half-planes 29
 - coefficient matrix, changing
 - component of 202–208, 209t
 - Coelli, T.J. 377–379, 383–384
 - commutative law
 - matrix algebra 14
 - vector algebra 20, 21
 - competition, perfect *see* perfect competition
 - complementarity, perfect 124
 - complementarity condition 329
 - complementary inputs 1, 2
 - complementary outputs 7
 - complementary pivot
 - method 329–335
 - complementary slackness
 - conditions 150–151, 189, 306, 324
 - complementary slackness
 - theorems 319–320
 - strong 104–106, 109–111
 - weak 102–104, 106, 109–111, 116
 - complementary solutions/
 - complementary basic solutions 329
 - composite activity 129, 132–133, 158
 - composite output activity 171
 - computational aspects 43–70
 - degenerate basic feasible solutions 66–69
 - dual simplex method 114–121
 - improving basic feasible solutions 48–65
 - simplex matrix 59–65, 68–70
 - simplex method 43–48, 69–70
 - constant product curves 125
 - see also* isoquants
 - constant returns to scale (CRS) 373
 - input-oriented DEA model under 387–390
 - constraint system 35
 - convex cone 30, 31f, 32
 - convex hull 33
 - convex polygons 174
 - convex polyhedral cone 31, 133
 - convex polyhedron 33, 34, 339
 - convex sets
 - boundary point 29
 - closed half-planes 29
 - cones 31–33
 - convex combination of X_1, X_2 , 29
 - hyperplane 29
 - interior point 29
 - linear form 29
 - and n -dimensional geometry 29–34
 - open half-planes 30
 - open or closed 29
 - quadratic programming 322
 - set of all convex combinations 33
 - spherical δ -neighborhood 29
 - strict separability 30
 - strictly bounded 29
 - supporting hyperplane 30
 - weak separation theorem 30

- convexification constraint 398
- Cooper, W.W. 338, 385–386, 398–402
- cost efficiency (CE) 383–384
- cost indifference curves *see* isocost curves
- cost minimization 7
 - and joint production 180–184
 - producing a given output 284–285
- costs
 - see also* cost efficiency (CE); cost indifference curves; cost minimization; marginal cost; total cost
 - average cost functions, determining 286–295
 - marginal *see* marginal cost
 - optimal dollar value of total cost 9
 - total imputed cost of firm's minimum output requirements 9
 - total potential cost reduction 9
- Craven, B. 338, 347–353
- critical values
 - parametric analysis 228, 236, 237, 242, 246, 249, 250
 - parametric programming and theory of the firm 262, 267, 269, 277
- CRS *see* constant returns to scale (CRS)

- d**
- Dano, S. 123, 139–146
- data envelopment analysis (DEA) 373–404, 374
 - see also* BCC (Banker, Charnes and Cooper) model; decision making units (DMUs)
 - allocative efficiency 380
 - best-practice extremal frontier 373
 - CCR (Charnes, Cooper and Rhodes) model 398, 400
 - constant returns to scale (CRS) 373
 - input-oriented DEA model under 387–390
 - convexification constraint 398
 - input and output slack variables 390–398
 - input distance function (IDF) 378–379
 - input-oriented 373, 387–390
 - envelopment form 389
 - isoquants 375
 - modeling 385–386
 - multiplier form 388
 - nonparametric 374
 - nonstochastic 374
 - output correspondence 375
 - output distance function (ODF) 377–378
 - output-oriented 373, 402–404
 - production
 - correspondence 386–387
 - projection path 373
 - set theoretic representation of a production technology 374–377
 - solving the BCC model 400
 - strong disposability 374, 375
 - technical efficiency 379, 380–383
 - technology set 374, 375
 - variable returns to scale (VRS) 373, 398–402
- Debreu, G. 380–383
- decision making units (DMUs) 373, 374, 377, 379
 - see also* data envelopment analysis (DEA)
 - degree of input-oriented technical efficiency 380–381

- efficient frontier 389
- fully efficient 382, 383
- input and output slack variables 393–397
- peer group 388
- reference set 388, 400
- synthetic 396, 397
- unit isoquant of fully efficient DMUs 380
- unit production possibility curve of fully efficient DMUs 381, 384
- degenerate basic feasible solutions 66–69
- demand function for a variable input 262–269
- diagonal matrix 14
- diminishing returns 135
- direct proportionality 2, 6–7
- distance 21
- distributive laws
 - matrix algebra 14, 15
 - vector algebra 20, 21
- divisibility, perfect *see* perfect divisibility
- DMUs *see* decision making units (DMUs)
- Dorfman, R. 123
- Dorn, W. 326
- Dreyfus, S. 315–320
- dual degeneracy 121
- dual feasibility *see* primal optimality (dual feasibility)
- dual problem
 - see also* duality theory; primal problem
 - artificial augmented form 113
 - dual quadratic programs 326–328
 - dual solution, constructing 106–113
 - duality theorems 103, 349, 353
 - generalized multiactivity profit-maximization model 161
 - joint production and cost minimization 184
 - as minimization (maximization) problem 95
 - optimal solutions 145, 184, 306, 399, 401
 - reformulation 297–310
 - simplex matrix 116
 - single-activity profit maximization model 141
- dual quadratic programs 325–328
- dual simplex method 113–114
 - addition of a new structural constraint 221, 222
 - basic feasible solutions 114, 117, 119
 - computational aspects 114–121
 - deletion of a variable 223
 - as an internalized resource allocation process 157
 - optimal solutions 114, 121, 122
 - summary 121–122
- dual solution, constructing 106–113
- dual structural constraints 183, 184, 338
- data envelopment analysis (DEA) 389
- dual solution, constructing 107, 108
- duality and complementary slackness theorems 320
- duality theorems 104, 105
- multiactivity profit maximization model 145
- reformulation of primal and dual problems 308
- simplex method 156
- single-activity profit maximization model 141
- symmetric duals 96

dual structural constraints (*cont'd*)
 unsymmetrical duals 99
 dual support cone 301
 duality theory 4, 95–122,
 297–370
see also dual problem; dual simplex
 method; dual structural
 constraints
 and complementary slackness
 theorems 315–320
 constructing the dual
 solution 106–113
 identity matrix *see* identity matrix
 Lagrangian saddle points 297,
 311–315
 in linear fractional functional
 programming 347–353
 optimal solutions 95, 106, 107,
 121, 122, 298, 302, 306,
 313–315
 duality theorems 101, 104
 pivot operations 114, 118, 120, 122
 primal 95
 reformulation of primal and dual
 problems 297–310
 simplex matrix 107–116, 118,
 119, 121
 symmetric duals 95–97, 98
 Taylor formula 316, 317
 theorems 100–106, 315–319,
 348–353
 unsymmetrical duals 97–100

e

echelon matrix 23, 24
 economic efficiency 126, 167
 economic rent 338
 efficiency
 allocative, cost, and
 revenue 383–384
 constant returns to scale
 (CRS) 388

economic 126, 167
 efficient subsets 375
 fully efficient DMUs 382, 383
 input efficient subsets 377
 output efficient subsets 375
 technical *see* technical efficiency (TE)
 unit isoquant of fully efficient
 DMUs 380
 unit production possibility
 curve of fully efficient
 DMUs 381, 384
 elementary row operation 16
 excess capacity 4
 existence theorem 101
 expansion path 126, 127, 294
 joint output 167, 169f
 expected payoff 357, 359
 extreme point solutions 39–40
 parametric analysis 231, 234
 extreme points 33, 43
 linear fractional functional
 programming 339–340
 parametric analysis 238, 239

f

factor learning
 learning economies 162
 learning index 163
 learning rates 162, 163
 negative exponential 163
 and optimum product mix
 model 164–165
 progress elasticity 163
 factor substitution 130
 Färe, R. 374–379
 Farrell, M.J. 380–383
 feasible directions 298
 feasible solutions
see also optimal solutions; primal
 optimality (dual feasibility);
 solutions
 basic/optimal basic

- addition of a new structural constraint 220, 221, 222
- degenerate 39, 66–69
- deletion of a structural constraint 223–224
- demand function for a variable input 262, 264–269
- determination of marginal productivity, average productivity and marginal cost 287–288, 290, 293
- dual simplex method 114, 117, 119
- dual solution, constructing 106, 108, 111
- improving 48–65
- inconsistency and redundancy 79–85
- linear fractional functional programming 340, 345, 352
- marginal (net) revenue productivity function for an input 271, 273, 274, 278, 279, 281, 284
- minimizing cost of producing a given output 285
- M*-Penalty method 73, 76, 77, 78
- new variable, addition of 217–219
- nondegenerate 40, 66
- parametric analysis 227–229, 231–233, 235–236, 239–243, 245, 248
- quadratic programming 324
- resource allocation with a fractional objective 354
- simplex method 155, 156
- supply function for the output of an activity 258–260
- two-phase method 90–93
 - updating the basis inverse 256
- definition 36
- dual quadratic programs 325, 326
- duality theorems 349
- extreme points 43
- linear complementarity problem (LCP) 329
- nonbasic 89
- profit indifference curves 148
- region of 35, 38, 148, 298
- Ferguson, C.E. 123, 139–146
- finite cone 30–31
- firm
 - technology of 123–125
 - theory of *see* theory of the firm
- fixed coefficients linear technology 157
- fixed inputs 4, 5, 11
 - see also* inputs
- data envelopment analysis (DEA) 384
- quadratic programming 335
- theory of the firm 155, 157, 159, 164
 - activity levels 150, 151
 - multiactivity profit maximization model 144, 145n
 - and parametric programming 257, 262, 269, 271, 275–277, 284, 290
 - single-activity profit maximization model 140, 142
- fixed resources 5, 6
 - see also* resources
- foregone profit 4, 5
- fractional objective, resource allocation with 353–356
- fractional programs 387
- Freimer, M. 315–320

Fried, H. 374–379
 Frisch, R. 181n
 fundamental theorems of linear programming 101, 102

g

game theory 356–363
 defining a game 356
 expected outcome 357
 expected payoff 357, 359
 fundamental theorem 359–360
 generalized saddle point 357
 matrix games 357–360
 transformation to a linear program 361–363
 mixed strategies 358, 359, 360, 363
 normal form 357
 outcome strictly dominated 357
 payoff function 357
 saddle point solution 358
 strategies 356, 358, 359, 360
 two-person games 356, 357
 value of the game 359
 zero-sum game 356
 Gauss elimination technique 24
 generalized multiactivity profit-maximization model 157–161, 335, 353
 generalized saddle point 359
 generalized saddle value 360
 gross profit 3
 gross profit margin 141, 144, 159, 161, 337, 338
 simplex-based optimization methods 337, 338, 355
 theory of the firm 145, 156, 157, 164
 and parametric programming 257, 262, 263, 269, 276–278, 281, 283–286

h

Hadar, J. 123, 139–146
 half-line 124
 homogeneity 21, 26, 27
 hyperplanes 29, 30, 339

i

identity matrix 14, 45n
 duality theory 108, 112
 M-Penalty method 72–75
 parametric programming 256
 sensitivity analysis 201
 inconsistency 78–85
 increasing (real resource)
 opportunity cost 175
 indifference curves
 cost 184–186, 190
 production 146, 147
 profit 146–148, 151–156
 infeasibility form 88
 input distance function (IDF) 378–379
 input isoquants 376–377
 inputs
 see also input distance function (IDF); input isoquants; outputs
 activities 1, 2
 complementary 1
 demand function for a variable input 262–269
 fixed *see* fixed inputs
 input activities 1–2
 input correspondence 376
 input efficient subsets 377
 input set 376
 input-conserving orientation 379, 380
 input-oriented BCC model 399
 input-oriented DEA model 387–390

- limitational 124
 - marginal (net) revenue
 - productivity function
 - for 269–276
 - optimal value of 5
 - shadow 180, 181n, 182
 - slack variables 390–398
 - strong disposability 375, 376
 - interior point 29
 - isocost curves 184–186, 190
 - surface 192f, 193
 - iso-input transformation
 - curve 167, 174
 - isoquants 125–127, 128f, 138, 375
 - see also* theory of the firm
 - ABCD 133, 134
 - input 376–377
 - joint process 131
 - multiple processes 133f
 - output 375
 - parametric representation 131
 - unit 131, 132, 380
 - “well-behaved,” 135
- j**
- joint output expansion path 167, 169f
 - joint output linear production
 - model 172
 - joint process isoquant 131
 - joint process linear production
 - model 130
 - joint process transformation
 - curve 172
 - joint product transformation curve
 - (iso-input) 167, 174
 - joint production 6
 - see also* production function; theory of the firm
 - and cost minimization 180–184
 - multiactivity joint-production
 - model 171–180, 177f, 179f
 - processes 165–166
- k**
- Karush-Kuhn-Tucker equivalence
 - theorem 313–315
 - Karush-Kuhn-Tucker necessary
 - and sufficient conditions 297
 - complementary pivot
 - method 329–330
 - quadratic programming 323, 327, 337
 - Karush-Kuhn-Tucker
 - theorem 303–310
 - corollaries 306–310
 - Kogiku, K.C. 123
 - Kornbluth, J. 347–353
 - Kuhn-Tucker-Lagrange necessary
 - and sufficient conditions 160
 - Kydland, F. 347–353, 355
- l**
- Lagrange technique 303
 - Lagrange multipliers 107, 160, 304, 306
 - Lagrangian expressions 107
 - Lagrangian saddle points 297, 311–315
 - linear fractional functional programming 350
 - Lasdon, L. 338
 - learning economies 162
 - learning index 163
 - learning rates 162, 163
 - Lemke, C.E. 113–114, 329–335
 - Liao, W. 164
 - limitationality 2, 7, 124, 166
 - mutual 126, 127
 - limiting subset 129
 - linear combinations 26–27
 - linear complementarity problem (LCP) 329
 - linear dependence 26–29
 - and linear independence 27
 - linear form 29, 321

linear fractional functional programming 338–346
 duality in 347–353
 linear model for the firm 1–2
 linear programming problem
 artificial 73
 augmented 36
 deletion of a structural constraint 223–224
 graphical solution to 37
 linear fractional programming 338
 new variable, addition of 217
 optimal solution to *see* optimal solutions
 sensitivity analysis 195–196
 surrogate 88, 90, 91, 93
 symmetric duals 95
 linear technology 123
 Lovell, C.A.K. 374–379, 383–386

m

Magnanti, T. 355
 marginal (net) revenue productivity
 function for an input 269–276
 marginal cost
 activity 189, 276–284, 282t
 determining 286–295
 imputed or shadow costs 9, 10
 joint production and cost minimization 183
 marginal (real) resource cost
 function 168, 169f, 179f
 marginal cost function for an activity 276–284
 marginal cost relationships 142
 multiactivity joint-production model 175n
 marginal product 136f, 175, 290, 291f
see also average product
 marginal product function 170
 marginal productivity

determining 286–295
 marginal productivity
 function 127, 138
 marginal profitability 156
 marginal revenue 142
 market prices 155
 Martos, B. 338, 355
 mathematical foundations
 convex sets and n -dimensional geometry 29–34
 linear dependence 26–29
 matrix algebra 13–20
 simultaneous linear equation systems 22–26
 vector algebra 20–22
 matrix
 algebra *see* matrix algebra
 augmented 23, 24
 basis matrix B 39, 49, 201, 203
 coefficient, changing component of 202–208, 209t
 defined 13
 diagonal 14
 echelon 23, 24
 identity 14, 73–75, 256
 n th order matrix A 18
 output technology 171
 premultiplier 15
 postmultiplier 15
 rank 23, 24
 simplex *see* simplex matrix
 submatrix 13, 25, 45
 transposition of 14
 triangular 14
 matrix algebra 13–20
see also matrix
 elementary row operation 16
 multiplication 15
 sweep-out process 18
 TYPE I operation 17
 TYPE II operation 17
 TYPE III operation 18
 matrix games 357–360

- transformation to a linear program 361–363
 - maximal-slack solution 390
 - method
 - complementary pivot 329–335
 - dual simplex *see* dual simplex method
 - M*-Penalty 71–78, 111, 294
 - simplex *see* simplex method; simplex-based optimization methods
 - two-phase 87–94
 - minimization of the objective function 85–86
 - Minkowski-Farkas theorem 302–303
 - mixed structural constraint system 71
 - Mond, B. 347–353
 - M*-Penalty method 71–78
 - basic feasible solutions 73, 76, 77, 78
 - dual solution, constructing 111
 - identity matrix 72–75
 - mixed structural constraint system 71
 - parametric programming 294
 - slack variables 71, 72, 76
 - surplus variables 71, 72, 76
 - multiactivity joint-production
 - model 171–180, 177f, 179f
 - see also* transformation curve
 - composite output activity 171
 - increasing (real resource)
 - opportunity cost 176
 - joint output linear production model 172
 - joint process transformation curve 172
 - joint product transformation curve 174
 - output technology matrix 171
 - parametric representation of the transformation curve 172
 - rate of product transformation 175
 - unit transformation curve 172, 173f
 - multiactivity production
 - function 129–139, 136f
 - additive activity 130
 - composite activity 130
 - diminishing returns 135
 - factor substitution 130
 - joint process linear production model 130
 - process substitution 130
 - technical rate of substitution 134–135
 - multiactivity profit maximization model 143–146
 - multiplicative identity 15, 20
 - mutual limitationality 126, 127
- n**
- Nanda, R. 162
 - Naylor, T. 123
 - n*-dimensional Euclidean space 21
 - negative exponential 163
 - nonbasic variables 39, 69, 118, 197, 212, 228
 - see also* basic variables
 - improving basic feasible solutions 50, 51, 54, 55
 - simplex method 44, 46, 47
 - simplex-based optimization methods 332, 340, 341
 - nondegeneracy assumption 44
 - nonnegativity conditions 35
 - nonreversible production activities 4
 - norm of X 21
 - normalizing constraints 387
 - Novaes, A. 345, 346
 - null vector 20, 27, 30, 98

O

- objective function 2, 3, 6, 8, 37, 43
 - see also* hyperplane; objective function coefficients
- artificial 74
- canonical forms 35
- computational aspects 46, 66, 67, 69, 70
 - improving basic feasible solutions 48, 49, 51, 52, 55, 57, 60, 61, 65
- deletion of a variable 223
- dual solution, constructing 110
- duality theorems 100, 102
- interpretation 269n
- minimization 85–86
- optimal 61, 388
- parameterizing 227, 228–236, 257
- primal 100, 106, 107, 223, 299, 307
- quadratic programming 321
- sensitivity analysis 198, 205
- surrogate 87–88
- two-phase method 88, 89
- objective function coefficients
 - 63, 164, 195, 217
 - see also* objective function changing 196–199, 209
 - parametric programming 229, 231, 259, 260
 - theory of the firm 182, 188
- operational level 124, 131
- opportunity cost 175
- optimal (imputed) costs of output quotas 10
- optimal (imputed) value of all fixed resources 5, 6
- optimal (imputed) value of outputs produced 10
- optimal accounting loss figures 5, 10
- optimal activity levels 4
- optimal criterion 51, 78, 85, 88, 92, 93, 113, 220
 - dual solution 107, 108, 112
 - inconsistency and redundancy 80, 81, 84
 - parametric programming 228, 237, 245, 249, 255
 - sensitivity analysis 196–198
- optimal dollar value of total cost 9
- optimal dollar value of total profit 4
- optimal objective function 61, 388
- optimal output activity levels 9
- optimal output configuration 9
- optimal primary-factor/labor-grade mix 8–9
- optimal product mix 4, 270
 - and factor learning 164–165
- optimal shadow price
 - configuration 5
- optimal simplex matrix *see* simplex matrix
- optimal solutions 4, 5, 10, 36–38, 40, 43, 67, 78, 202
 - see also* feasible solutions; solutions canonical forms 36
 - data envelopment analysis (DEA) 388, 390, 397, 399–402
 - dual problem 145, 184, 306, 399, 401
 - duality theory 95, 106, 107, 298, 313–315
 - dual simplex method 114, 121, 122
 - reformulation of primal and dual problems 302, 306
 - theorems 101, 104
- existence and location 38–39
- parametric programming 241, 246, 265, 271, 272
- quadratic programming 322–324
- simplex-based optimization
 - methods 322, 326, 328, 337, 338, 346, 347, 349, 352, 355, 363
- structural changes 217, 222, 223f

- theory of the firm 141, 145, 184
- optimal utilization information 4
- optimal value of inputs 5
- optimality evaluators 48
- optimality theorem 48, 51
- optimum product-mix model 164–165
- output activities 6, 7, 183–185, 187, 190, 191
 - multiactivity joint-production model 171–174
 - output activity mix 189
 - output activity vector 165, 174
- output distance function (ODF) 377–378
- output efficient subsets 375
- output process ray 166
- output substitution 172
- output technology matrix 171
- output transformation curves 174, 175, 176f, 184–186
- outputs
 - see also* inputs; output activities; output distance function (ODF); output efficient subsets; output process ray; output substitution; output technology matrix; output transformation curve
 - cost minimization 284–285
 - fixed level 131
 - joint output expansion path 167, 169f
 - joint output linear production model 172
 - optimal (imputed) value of outputs produced 10
 - optimal imputed costs of output quotas 10
 - optimal output activity levels 9
 - optimal output configuration 9
 - output correspondence 375
 - output set 375
 - output-augmenting orientation 380
 - output-oriented DEA 377–378, 402–404
 - output-oriented multiplier problem 403
 - quotas 10, 167, 175, 187
 - slack variables 390–398
 - supply function for the output of an activity 257–262
 - total imputed cost of firm's minimum output requirements 9
 - transformation surface 190, 191f
 - unit level 132, 354
 - overproduction 9, 182, 189
- p**
 - Panik, M. 123, 351
 - parallelogram law, vector addition 132
 - parametric analysis 11, 227–256
 - see also* parametric programming and theory of the firm
 - basis inverse, updating 256
 - critical values 228, 236, 237, 242, 246, 249, 250
 - parameterizing an activity vector 245–256
 - parameterizing the objective function 227, 228–236, 257
 - parameterizing the requirement vector 236–245, 277
 - primal feasibility 228, 236, 237, 245, 249, 250, 253
 - revised feasibility criterion 237
 - revised optimality condition 228, 245, 249
 - parametric programming and theory of the firm 257–295
 - see also* parametric analysis
 - average cost functions, determining 286–295
 - average productivity, determining 286–295

- parametric programming and
 - theory of the firm (*cont'd*)
 - ceteris paribus assumption 257, 262, 269, 276, 290, 292
 - critical values 262, 267, 269, 277
 - demand function for a variable input 262–269
 - marginal (net) revenue productivity function for an input 269–276
 - marginal cost,
 - determining 286–295
 - marginal cost function for an activity 276–284, 282t
 - marginal productivity,
 - determining 286–295
 - minimizing cost of producing a given output 284–285
 - supply function for the output of an activity 257–262
 - parametric representation of the isoquant 131
 - parametric representation of the transformation curve 172
 - payoff function 357
 - perfect competition 1, 6
 - generalized multiactivity profit-maximization model 158
 - multiactivity profit maximization model 143–144
 - single-activity profit maximization model 140, 142
 - perfect complementarity 124, 166
 - perfect divisibility 1, 6, 124, 166
 - pivot operations 74, 89
 - complementary pivot method 332, 334
 - computational aspects 52, 56, 70
 - dual simplex 121, 157, 253, 271, 274
 - duality theory 114, 118, 120, 122
 - parametric programming 239, 253, 259, 260, 264, 265, 271, 274
 - pivotal term 52
 - plane of support theorem 30–31
 - polar support cone 300
 - postmultiplier matrix 15
 - post-optimality analysis 10, 195
 - premultiplier matrix 15
 - primal feasibility 220, 280
 - duality theory 114, 120, 122
 - parametric analysis 228, 236, 237, 245, 249, 250, 253
 - sensitivity analysis 200, 206–208, 209t, 210, 213, 214
 - primal objective function 100, 106, 107, 223, 299, 307
 - primal objective value 121, 325
 - primal optimality (dual feasibility)
 - see also* sensitivity analysis
 - changes in technology 214, 215
 - changing a component of the coefficient matrix 203, 204, 206–208
 - changing a component of the requirements vector 200, 202t
 - changing objective function coefficient 196, 198, 199t
 - changing product and factor prices 212–213
 - changing resource requirements 213
 - parametric analysis 245, 246, 250
 - primal problem 4
 - see also* dual problem; duality theory
 - canonical form 95, 97
 - dual quadratic programs 325–327
 - dual simplex method 114, 115
 - dual solution, constructing 107
 - duality theorems 100, 102, 103, 306, 348, 349
 - generalized short-run fixed-coefficients profit-maximization model 159–160
 - Lagrangian saddle points 311
 - as maximization (minimization) problem 95

- reformulation 297–310
 - structural constraints 96, 99
 - symmetric duals 95–97
 - unsymmetrical duals 98, 99
 - primal simplex matrix 108, 114, 118, 201
 - primal simplex method 113
 - primary-factor, optimal 8–9
 - principal diagonal 14
 - problems
 - artificial 75–77
 - dual *see* dual problem
 - linear complementarity problem (LCP) 329
 - linear programming *see* linear programming problem
 - optimization 284, 286
 - output-oriented multiplier 403
 - parametric 258, 260–262, 265–267, 269, 270
 - primal *see* primal problem
 - primal maximum 297
 - profit maximization 149
 - reformulation of primal and dual problems 297–310
 - saddle point 297, 312
 - symmetrical 284
 - process ray 124
 - process substitution 130, 172
 - product mix, optimal 4, 164–165, 270
 - product transformation curve 177f
 - joint product 167, 174
 - “well-behaved” product 175
 - product transformation function, single-process 167–170, 169f
 - production activities 1, 4, 6, 123
 - production correspondence 386–387
 - production function
 - see also* joint production; theory of the firm
 - joint production process 165–166
 - multiactivity 129–139, 136f
 - single-process 125–127, 128f, 129
 - production indifference curves 146, 147
 - production possibility set 374, 386
 - production time 354
 - profit
 - see also* profit indifference curves; profit maximization model; profit maximization problem
 - average 2, 354
 - foregone 4, 5
 - gross *see* gross profit
 - objective function 3
 - total *see* total profit
 - profit indifference curves 146–148, 151–156
 - profit maximization model
 - assumptions underlying 1–2
 - generalized multiactivity 157–161, 335, 353
 - multiactivity 143–146
 - short-run fixed-coefficients 140–141
 - short-run linear technology 144
 - simplex-based optimization methods 335, 336
 - single-activity 139–142
 - profit maximization problem 149
 - progress elasticity 163
 - proportionality, direct 2, 6–7
- q**
- quadratic forms 321, 363–371
 - classification 367–368
 - definite 367, 368–370
 - general structure 363–365
 - indefinite 367
 - necessary and sufficient conditions for the definiteness and semi-definiteness of 369–370

- quadratic forms (*cont'd*)
 - necessary conditions for
 - definiteness and semi-definiteness of 368–369
 - semi-definite 367, 368–370
 - symmetric 366–367
 - theorems 368–371
 - quadratic programming 321–324
 - and activity analysis 335–338
 - dual programs 325–328
 - Karush-Kuhn-Tucker necessary and sufficient conditions 324, 327
 - primal programs 325
 - theorems 325–328
 - quasi-rents 338n
- R**
- rank 22, 23
 - rate of product transformation 175
 - redundancy 39, 78–85, 89, 126
 - Reeves, G. 164, 165
 - reference set 388, 400
 - requirements space 41, 294
 - requirements vector 41
 - changing component of 200–202, 209–210
 - determination of marginal productivity, average productivity and marginal cost 286–290
 - marginal (net) revenue productivity function for an input 272–274
 - parameterizing 236–245, 277
 - resources
 - activity levels interpreted as individual resource levels 186–193
 - allocation process
 - dual simplex method 157
 - fractional objective, resource allocation with 353–356
 - simplex method 155–156
 - average (real) resource cost function 168, 179f
 - changing resource requirements 213
 - fixed 5, 6
 - level of utilization 166, 167, 185
 - marginal (real) resource cost function 168, 179f
 - optimal utilization information 4
 - optimal valuation of the firm's fixed resources 5
 - resource requirements vector 124, 158
 - total (real) resource cost function 168, 169f, 179f
 - total imputed value of firm's fixed resources 5
 - returns to scale
 - BCC (Banker, Charnes and Cooper) model 401–402
 - constant 373
 - data envelopment analysis (DEA) 401–402
 - technology of the firm 123, 124
 - variable 373
 - revenue efficiency (RE) 383–384
 - robustness 10
- S**
- Saaty, T. 326
 - saddle points
 - game theory 358, 359
 - generalized 359
 - Lagrangian *see* saddle points, Lagrangian
 - saddle points, Lagrangian 297, 311–315
 - problem 297, 312
 - theorems 312–315
 - necessary and sufficient condition 312–313
 - sufficient condition 313
 - Salkin, G. 347–353
 - scalar (inner) product 20

- Schnaible, S. 347–353
- Seiford, L.M. 373–374
- sensitivity analysis 10, 195–215
 - changes in technology 213–215
 - changing a component of an activity vector 210–215
 - changing a component of the coefficient matrix 202–208, 209t
 - changing a component of the requirements vector 200–202, 209–210
 - changing product and factor prices 211–213
 - changing resource requirements 213
 - objective function coefficient, changing 196–199, 209
 - post-optimality analysis 10, 195
 - primal feasibility 200, 206–208, 209t, 210, 213, 214
 - simplex matrix 195, 196, 198, 201, 205, 213, 214
 - summary of effects 209–215
- shadow activities 181
- shadow inputs 180, 181n, 182
- shadow prices 4
 - dual simplex method 157
 - optimal shadow price configuration 5
 - resource allocation with a fractional objective 355
 - single-activity profit maximization model 140
- Shephard, R.W. 377–379
- short run
 - firm operating in 2
 - generalized short-run fixed-coefficients profit-maximization model 159
 - short-run fixed-coefficients profit-maximization model 140–141
 - short-run linear technology profit-maximization model 144
 - short-run supply curve 142
- simple activity 123, 130
- simplex 34
- simplex matrix 50–57, 331
 - computational aspects 59–65, 68–70
 - duality theory 107–116, 118, 119, 121
 - optimal
 - computational aspects 61, 63
 - duality theory 109, 110, 113, 294
 - parametric analysis 227, 230–234, 238–245, 247, 248, 251, 253, 255
 - parametric programming and theory of the firm 258, 259, 263, 270, 277, 278, 285, 286
 - redundancy 79
 - sensitivity analysis 195, 196, 198, 201, 205, 213, 214
 - simplex-based optimization methods 345, 346, 352, 353, 355
 - structural changes 217, 218, 221, 223–225
 - theory of the firm 149, 188
- parametric programming 227, 263, 270, 285, 286, 289, 294
 - activity vector 247, 248, 251, 253, 255
 - marginal cost function for an activity 277, 278
 - parameterizing the objective function 229–235
 - parameterizing the requirement vector 238–245
 - supply function for the output of an activity 258, 259
- primal 108, 114, 118, 201
- sensitivity analysis 195, 196, 198, 201, 205, 213, 214
- simplex-based optimization methods 342, 343, 345, 346, 352, 353, 355

- simplex matrix (*cont'd*)
 - structural changes 217–225
 - theory of the firm 149, 188
 - variations of standard simplex
 - routine 72, 74–76, 79–83, 88–93
- simplex method 4, 43–48
 - see also* simplex-based optimization methods
 - dual *see* dual simplex method
 - dual solution, constructing 111
 - as an internal resource allocation process 155–156
 - nondegeneracy assumption 44
 - primal 113
 - summary 69–70
 - symmetric duals 95
 - variations of standard simplex
 - routine 71–94
- simplex-based optimization
 - methods 321–371
 - see also* simplex method
 - complementary pivot method 329–335
 - duality in linear fractional functional
 - programming 347–353
 - game theory 356–363
 - linear fractional functional programming 338–346
 - matrix games 357–360
 - optimal solutions 322, 326, 328, 337, 338, 346, 347, 349, 352, 355, 363
 - quadratic forms 363–371
 - quadratic programming 321–324
 - and activity analysis 335–338
 - dual quadratic programs 325–328
 - resource allocation with a fractional objective 353–356
 - simplex matrix 342, 343, 345, 346, 352, 353, 355
- simultaneous linear equation systems 22–26
 - consistency 22
 - determinate solutions 26
 - homogenous 26, 27
 - n linear equations in n unknowns 22
 - rank 22, 23
 - theorems 23–26
 - underdetermined 25
- single-activity profit maximization model 139–142
- single-process product transformation function 167–170, 169f
- single-process production function 125–129, 128f
 - average productivity function 127
 - expansion path 126, 127
 - limiting subset 129
 - marginal productivity function 127
- slack variables 82, 189, 303, 403
 - see also* basic variables; complementary slackness theorems; nonbasic variables; variables
 - dual solution 112
 - duality theorems 103, 105
 - input and output 390–398
 - M -Penalty method 71, 72, 76
 - nonnegative 36, 45, 67, 71, 109, 149, 160, 211, 285, 342
 - dual simplex method 115, 116
 - duality theory 303, 306
 - improving basic feasible solutions 57, 59, 62, 64
 - linear fractional functional programming 352, 353
 - structural changes 219, 222, 224
 - primal 103–105, 112, 145, 306, 319, 320
 - simplex-based optimization methods 320, 340, 353
 - structural changes 219, 221, 222, 224

- theory of the firm 150, 156, 160, 161, 183, 184
- solutions
 - admissible 298
 - almost complementary
 - basic 330–332
 - basic 28
 - canonical forms 36
 - complementary/complementary
 - basic 329
 - dual, constructing 106–113
 - extreme point 39–40, 43
 - feasible *see* feasible solutions
 - graphical, to linear programming problem 37
 - maximal-slack 390
 - optimal *see* optimal solutions
 - and requirements spaces 41
 - saddle point 358
 - zero-slack 390
- solutions space 41
- spaces
 - n -dimensional Euclidean 21
 - requirements 41, 294
 - solutions 41
 - vectors 20, 28
- spanning set, vectors 27
- spherical δ -neighborhood 29
- standard forms 36
- static models 1, 6
- strategies, game theory 356
 - maximin 360
 - minimax 360
 - mixed 358, 359, 360, 363
- strict separability 30
- strong complementary slackness
 - theorems 104–106, 109–111
- strong disposability 374, 375, 376
- structural changes 11, 217–226
 - addition of a new structural constraint 219–222
 - addition of a new variable 217–219
 - deletion of a structural constraint 223–226
 - deletion of a variable 223
 - optimal solutions 217, 222, 223f
 - simplex matrix 217–225
- structural constraints 4, 5, 9
 - activity levels 150
 - addition of 219–222
 - artificial augmented structural constraint system 72
 - augmented structural constraint system 36, 39, 53
 - primal 155–156
 - canonical forms 35
 - complementary pivot method 329
 - deletion of 223–226
 - dual 183, 184, 338
 - data envelopment analysis (DEA) 389
 - dual solution,
 - constructing 107, 108
 - duality and complementary slackness theorems 320
 - duality theorems 104, 105
 - multiactivity profit maximization model 145
 - reformulation of primal and dual problems 308
 - simplex method 156
 - single-activity profit maximization model 141
 - symmetric duals 96
 - unsymmetrical duals 99
 - generalized multiactivity profit-maximization model 161
 - inconsistency and redundancy 75
 - inequality 97, 302
 - linear fractional functional programming 340, 352
 - original system 78, 79, 85
 - primal problem 96, 99
 - reformulation of primal and dual problems 307
 - sensitivity analysis 202
 - single-activity profit maximization model 142

- submatrix 13, 25, 45
- substitution
 - factor 130
 - output 172
 - process 130, 172
 - technical rate of 134–135, 154
- sum vector 1 21
- supply function for the output of an activity 257–262
- supporting hyperplane 30
- surplus variables 86, 306, 320, 353
 - see also* basic variables; nonbasic variables; slack variables; variables
 - duality theory 96, 103, 104, 105, 108, 112–114, 119
 - inconsistency and redundancy 79, 82
 - M*-Penalty method 71, 72, 76
 - nonnegative 71, 188, 221, 353
 - structural changes 221–226
 - theory of the firm 141, 145, 150, 157, 182, 184
 - activity levels 188, 189
- surrogate linear programming problem 88, 90, 91, 93
- surrogate objective function 87–88
- Swarup, K. 340, 342
- Sweigart, J. 164, 165
- symmetric duals
 - duality theory 95–97, 98
 - joint production and cost minimization 183
 - theory of the firm 141, 144, 148, 161
- t**
- tangent support cone 298
- technical efficiency (TE) 126, 132, 167, 379, 389
 - degree of input-oriented technical efficiency 380–381
 - degree of radial technical inefficiency 381, 382
 - input-oriented measure 381
 - measuring 380–383
 - output-oriented measure 382
 - projection points 382
 - radial measures 380, 381, 382
 - technically efficient projection point 381
- technical rate of substitution 134–135, 154
- technological changes 213–215
- technological independence 165
- technological interdependence 6, 165
- technology of the firm 123–125
- technology set 374, 375
- theorems
 - basic feasible solutions 40
 - complementary slackness *see* complementary slackness theorems
 - convex sets 30–34
 - duality 100–106, 315–319, 348–353
 - existence 101
 - fundamental, of linear programming 101, 102
 - game theory 359–360
 - Karush-Kuhn-Tucker 303–310
 - Karush-Kuhn-Tucker equivalence 313–315
 - minimization of the objective function 86
 - Minkowski-Farkas 302–303
 - necessary and sufficient condition 312–313
 - optimal solutions 38–39
 - optimality 48, 51
 - plane of support 30–31
 - quadratic forms 368–371
 - quadratic programming 325–328
 - reformulation of primal and dual problems 299–310
 - saddle points, Lagrangian 312–315

- simultaneous linear equation systems 23–26
- sufficient condition 313
- weak separation 30
- theory of the firm 123–193
 - activity levels interpreted as individual product levels 148–155
 - activity levels interpreted as individual resource levels 186–193
 - cost indifference curves 184–186
 - dual simplex method 157
 - factor learning and optimum product-mix model 161–165
 - generalized multiactivity profit-maximization model 157–161
 - isoquants *see* isoquants
 - joint production
 - and cost minimization 180–184
 - multiactivity joint-production model 171–180, 177f, 179f
 - processes 165–166
 - multiactivity production
 - function 129–139, 136f
 - multiactivity profit maximization model 143–146
 - optimal solutions 141, 145, 184
 - and parametric programming *see* parametric programming and theory of the firm
 - profit indifference curves 146–148, 151–156
 - simplex method 155–156
 - single-activity profit maximization model 139–142
 - single-process product transformation
 - function 167–170, 169f
 - single-process production
 - function 125–127, 128f, 129
 - technology of the firm 123–125
- Thompson, G.E. 123
- Thrall, R.M. 373–374, 385–386, 398–402
- total cost
 - imputed cost of all output requirements 10
 - imputed cost of firm's minimum output requirements 9
 - joint production and cost minimization 183
 - marginal cost function for an activity 276, 281, 283
 - optimal dollar value of 9
 - total (real) resource cost function 168, 169f, 179f
 - total conversion cost 158
 - total variable cost (TVC) 263, 292–294
- total factor productivity 387
- total profit 2–6, 211, 295, 338, 354
 - activity levels 149, 150
 - optimal dollar value of 4
 - profit indifference curves 146–148
 - simplex method 155
 - single-activity profit maximization model 140
 - theory of the firm 140, 146–150, 155
- total product function 127, 170
- transformation curves 167
 - joint process 172
 - joint product 167, 174
 - output 174, 175, 176f, 184–186
 - parametric representation 172
 - unit 172, 173f, 174
 - “well-behaved” product 175
- transposition, matrix 14
- triangular inequality 21
- triangular matrix 14
- two-person games 356, 357
- two-phase method 87–94
 - infeasibility form 88
 - input and output slack variables 390

two-phase method (*cont'd*)
 surrogate linear programming
 problem 88
 surrogate objective function
 87–88

U

unit column vector 21
 unit isoquant 131, 132, 380
 unit level of activity 124
 unit transformation curve 172,
 173f, 174
 unrestricted variables 86–87
 unsymmetrical duals 97–100

V

Vandermullen, D. 123
 variable returns to scale (VRS) 373
 modeling 398–402
 variables
 addition of a new variable
 217–219
 artificial *see* artificial variables
 basic *see* basic variables
 deletion of 223
 demand function for a variable
 input 262–269
 legitimate 72, 79, 88
 nonbasic *see* nonbasic variables
 slack *see* slack variables
 surplus *see* surplus variables
 unrestricted 86–87
 vector algebra 20–22
see also activity vector; vectors
 vector space 20, 28

vectors

activity *see* activity vector
 algebra *see* vector algebra
 artificial 73, 75, 78, 79, 81, 82,
 91, 93
 components 20
 definition 20
 nonbasic 202, 203, 256
 null vector 20, 27, 30, 98
 orthogonal 21
 output activity 165, 174
 requirements *see* requirements
 vector
 resource requirements 124, 158
 spanning set 27
 sum vector 1 21
 unit column 21
 Vernon, J. 123
 vertex (of a cone) 30
 vertex (of a convex set) 33

W

Wagner, H. 347–353
 weak complementary slackness
 theorem 102–104, 106,
 109–111, 116
 weak disposability 376
 weak separation theorem 30

Y

Yuan, J. 347–353

Z

zero-slack solution 390
 zero-sum game 356