Subject Index

A

Added variable plot 5, 61, 76 Akaike's information criterion, AIC 10,37 Algorithms 133-148 Gaussian elimination 140 likelihood-based 133-135 1D algorithm 141, 257–262 component screening (ECS) 142 - 145coordinate decent (ECD) 142 non-Grassmann 139-141 sequential 141-145 starting values 135–139 moment-based 145-148, 262-266 nested envelope estimators 102, 141, 145 selecting constituent estimators 135 SIMPLS, see Partial least squares Asymptotic standard error 27

В

Bayes information criterion, BIC 10, 37 Bingham distribution 172 Bonferroni inequality 17, 66 Box's M test 129 Box–Cox power transformation 40

С

Canonical correlations 66, 191–194 connection to envelopes 194 review 191-194 Canonical variates 191 Canonical vectors 191 Central subspace, see Sufficient dimension reduction Compound symmetry 13 CORE, see Sufficient dimension reduction for covariance matrices Covariance matrices, comparison of and envelopes 215 SDR methods 213-215 spectral methods 212-213

D

Dimension reduction subspace, DRS, see Sufficient dimension reduction Dimension selection asymptotic behavior 38–41 bounding the rank 38 model free 141 overestimation 41–43 underestimation 41–43 via cross validation 37 via information criteria 37

An Introduction to Envelopes: Dimension Reduction for Efficient Estimation in Multivariate Statistics, First Edition. R. Dennis Cook. © 2018 John Wiley & Sons, Inc. Published 2018 by John Wiley & Sons, Inc.

288 Subject Index

Dimension selection (*Continued*) via likelihood ratio testing 36via likelihood ratio testing, LRT(α) 36, 37

Ε

Eigenspace, see Matrix algebra, eigenspace Envelope algebra 240-246 coordinate reductions 244-246 direct sums 244 envelope relationships 241-244 Envelopes asymptotic, definition of 150 basic 7 Bayesian 171-172 characterization 8 for a matrix-valued parameter 157 for a matrix-valued response 160-166 for a vector-valued parameter 149 - 157heteroscedastic 126-131 definition 127 regression 130-131 inner 173-182 definition 174 inner response envelopes 175 relationship to response envelopes 175 predictor, see Predictor envelopes reduced-rank predictor 199 reduced-rank response 197-199 response, see Response envelopes scaled predictor 187-190 scaled response 182-187 sparse 33, 103, 145, 168-171 when r > n 168–170 when $r \ll n$ 170–171 spatial 166-167 tensor 157 definition 159 Extreme components 225

F

Functional data 126, 211-212

G

Grassmannian 20, 24, 71, 142, 171 algebraic dimension 20 derivatives 251–252 optimization 267–272

I

Illustrations Air pollution 59–63 added variable plots 62 envelope estimate of Σ 63 immaterial variation, plot of 63 response envelopes 61-62 Aster models 156-157 Australian Institute of Sport predictor envelopes 103–105 response envelopes 58 - 59Banknotes comparing covariance matrices 216 response envelopes 54-55 Berkeley Guidance Study bootstrap performance 53 response envelopes 51-54 Birds-planes-cars comparing covariance matrices 216 - 217Brain volumes 65-67 plots of canonical variates 66 sparse fit 169 Cattle weights bootstrap smoothing 49 bootstrapping u 47–48 heteroscedastic envelope fit 129 influence of *u* 43 initial envelope fit 14partial response envelopes 74 sparse fit 169 *X*-invariant part of *Y* 23 Egyptian skulls 55–58 response envelopes 56

Meat properties 109 Mens' urine added variable plot 76 partial response envelope 76 response transformations 75 Minneapolis schools transformed responses 123 untransformed responses 124 Multivariate bioassay 63–65 Mussels' muscles predictor envelopes 106-109 sliced inverse regression 205 - 206Pulp fibers partial response envelopes 78 partial response envelopes for prediction 78 response envelopes 78 Race times inner envelopes 179-182 scaled response envelopes 185 - 187Wheat protein bootstrap 46 full data response envelope 51 introductory illustration 13 predicting protein content 105 Invariant subspace 235-236, 238 definition 235 Inverse Gaussian distribution 172

Κ

Krylov matrices 90, 101, 147, 207, 210

L

Longitudinal data 6, 13, 79, 117 LRT(α), *see* Dimension selection

М

Majorization-minimization principle 169Matrix algebra *vec* and *vec* h 246–247

commutation matrices 248-249 derivatives 249-251 determinants 252-253 eigenspace 237-238, 241-244 definition 237 expansion and contraction matrices 248 - 249Kronecker products 246–247 matrix exponential 268 similar matrices 183 skew symmetric matrix 268 Matrix normal distribution 159, 163, 166, 255 - 256Multivariate linear model 2 asymptotic properties 5 estimation 2-4 Fisher information 5 OLS estimator 2 standardized OLS estimator 3 Z-score 4, 14, 15 Multivariate mean envelope 151 Multivariate mean envelopes 117-131 multiple means 126-130 single mean 117-126

Ν

NIPALS algorithm, see Partial least squares Non-linear least squares 154

Ρ

Parallel coordinate plot 15 Partial envelopes for prediction 77–78 Partial least squares 81 latent variable formulation 84–86 NIPALS algorithm 88 scaled SIMPLS algorithm 189–190 SIMPLS algorithm 88, 90, 105, 109, 145, 173, 210 Krylov matrices and 90 properties of when n < p 91–94

290 Subject Index

Partial least squares (*contd*.) response scale, importance of 98 sparse version 91 vs OLS 90 sparse versions 92, 94 Partial predictor envelopes 152 - 153Partial response envelopes asymptotic distribution 72 71-72 estimation model 69 rationale 69, 152 schematic illustration 70 selecting u 73 PCR, see Principal component regression Penalized objective functions 168-170 PFC, see Principal fitted components PLS, see Partial least squares Prediction Wheat protein data 105 with partial response envelopes 77 - 78with response envelopes 28-29 Predictor envelopes dimension selection 101 functional 211-212 likelihood based estimation 95–96 asymptotic properties 98-100 fitted values and predictions 100 vs principal components 98 vs SIMPLS 98 model 83 motivation for 81-83, 151 parameter count 83 potential advantages 86-88 reduced-rank, see Envelopes, reduced-rank scaling the predictors 187-190 Predictor-response envelopes 109 - 115

asymptotic properties 115 estimation 113-115 model 109 potential gain 110-113 Principal component regression 25, 82,98 Principal components 62, 85 Principal fitted components 229-233 and SIR 233 envelopes and 230, 231 high-dimensional 233 model 229 model choices 232-233 non-normal errors 232 with anisotropic errors 231 with isotropic errors 230-231 Probabilistic principal components 200, 219-228 asymptotic behavior 226-228 envelope formulations 220 - 225225 dimension selection intrinsic/extrinsic variation 220 with isotropic intrinsic variation 223-225 with isotropic variation 222-223 fixed latent model 225 isotropic variation 219 random latent model 219 Profile plots 15, 16, 18, 44, 45

R

Reduced-rank regression 195–199 contrast with envelopes 196 Reducing subspace 7–9, 235–240 definition 7, 235 Reflexive indicator 85 Relevant components 101 Response envelope partial, *see* Partial response envelopes Response envelopes asymptotic distributions 25–28 bootstrap 45-50 bootstrap smoothing 48 - 50clues to effectiveness 14 definition 8 diagnostic plots 44, 63 dimension selection, see Dimension selection fitted values 28 heteroscedastic regressions 130 - 131immaterial variation and X-invariants 7 inner, see Envelopes, inner introduction 6-10 introductory illustrations 10-18 maximum likelihood estimation 21 - 23maximum likelihood estimators 23 model 8 motivation 7,152 multivariate mean, see Multivariate mean envelopes non-normal errors 34 parameter count 19 partial, see Partial response envelopes potential advantages of 21 prediction, see Prediction reduced-rank, see Envelopes, reduced-rank scaled, see Envelopes, scaled scaling the responses 25, 182–187 sparse, see Envelopes, sparse sufficiency and 19 testing responses 29-34 X-invariants 7 Z-score 27

_

S

SDR, see Sufficient dimension reduction SIMPLS algorithm, see Partial least squares SIR, see Sliced inverse regression Sliced inverse regression 204 - 207envelopes and 207 Sparse permutation invariant covariance estimation 170 - 171Stiefel manifold 171 Sufficient dimension reduction 155, 202 - 204central mean subspace, definition 207 - 208central subspace 202-203, 231 definition 202 conditional mean reduction 207 - 211and envelopes 209-211 and SIMPLS 210 envelopes and 209 iterative Hessian transformations 210connection to envelopes 204 constant covariance condition 203 dimension reduction subspace 202 for covariance matrices 212-217 and envelopes 215 functional predictor 211 linearity condition 155, 203 Supervised singular value decomposition 199-201

W

Weighted least squares 153-154